Binomial Theorem

Question 1.
The number $(101)^{100} - 1$ is divisible by
(a) 100
(b) 1000
(c) 10000
(d) All the above

Answer: (d) All the above
Given, $(101)^{100} - 1 = (1 + 100)^{100} - 1$
$= [{}^{100}C_0 + {}^{100}C_1 \times 100 + {}^{100}C_2 \times (100)^2 + \ldots\ldots\ldots + {}^{100}C_{100} \times (100)^{100}] - 1$
$= 1 + [{}^{100}C_1 \times 100 + {}^{100}C_2 \times (100)^2 + \ldots\ldots\ldots + {}^{100}C_{100} \times (100)^{100}] - 1$
$= {}^{100}C_1 \times 100 + {}^{100}C_2 \times (100)^2 + \ldots\ldots\ldots + {}^{100}C_{100} \times (100)^{100}$
$= 100 \times 100 + {}^{100}C_2 \times (100)^2 + \ldots\ldots\ldots + {}^{100}C_{100} \times (100)^{100}$
$= (100)^2 + {}^{100}C_2 \times (100)^2 + \ldots\ldots\ldots + {}^{100}C_{100} \times (100)^{100}$
$= (100)^2 [1 + {}^{100}C_2 + \ldots\ldots\ldots + {}^{100}C_{100} \times (100)^{98}]$
Which is divisible by 100, 1000 and 10000

Question 2.
The value of -1° is
(a) 1
(b) -1
(c) 0
(d) None of these

Answer: (b) -1
First we find 10
So, 10 = 1
Now, -10 = -1

Question 3.
If the fourth term in the expansion $(ax + 1/x)^n$ is 5/2, then the value of x is
(a) 4
(b) 6
(c) 8
(d) 5

Answer: (b) 6
Given, $T_4 = 5/2$
$\Rightarrow T_{3+1} = 5/2$
$\Rightarrow {}^nC_3 \times (ax)^{n-3} \times (1/x)^3 = 5/2$
$\Rightarrow {}^nC_3 \times a^{n-3} \times x^{n-3} \times (1/x)^2 = 5/2$
Clearly, RHS is independent of x,
So, $n - 6 = 0$
$\Rightarrow n = 6$

Question 4.
The number 111111 ………….. 1 (91 times) is
(a) not an odd number
(b) none of these
(c) not a prime
(d) an even number

Answer: (c) not a prime
111111 ………….. 1 (91 times) = 91 × 1 = 91, which is divisible by 7 and 13.
So, it is not a prime number.

Question 5.
In the expansion of $(a + b)^n$, if n is even then the middle term is
(a) $(n/2 + 1)^{th}$ term
(b) $(n/2)^{th}$ term
(c) n^{th} term
(d) $(n/2 - 1)^{th}$ term

Answer: (a) $(n/2 + 1)^{th}$ term
In the expansion of $(a + b)^n$
if n is even then the middle term is $(n/2 + 1)^{th}$ term

Question 6.
The number of terms in the expansion $(2x + 3y - 4z)^n$ is
(a) n + 1
(b) n + 3
(c) {(n + 1) × (n + 2)}/2
(d) None of these

Answer: (c) {(n + 1) × (n + 2)}/2
Total number of terms in $(2x + 3y - 4z)^n$ is
$= {}^{n+3-1}C_{3-1}$
$= {}^{n+2}C_2$
= {(n + 1) × (n + 2)}/2

Question 7.
If A and B are the coefficient of x^n in the expansion $(1 + x)^{2n}$ and $(1 + x)^{2n-1}$ respectively, then A/B equals
(a) 1
(b) 2
(c) 1/2
(d) 1/n

Answer: (b) 2
$A/B = {}^{2n}C_n / {}^{2n-1}C_n$
= {(2n)!/(n! × n!)}/{(2n – 1)!/(n! × (n – 1!))}
= {2n(2n – 1)!/(n(n – 1)! × n!)}/{(2n – 1)!/(n! × (n – 1!))}
= 2
So, A/B = 2

Question 8.
The coefficient of y in the expansion of $(y^2 + c/y)^5$ is
(a) 29c
(b) 10c
(c) $10c^3$
(d) $20c^2$

Answer: (c) $10c^3$
We have,
$T_{r+1} = {}^5C_r \times (y^2)^{5-r} \times (c/y)^r$
$\Rightarrow T_{r+1} = {}^5C_r \times y^{10-3r} \times c^r$

For finding the coefficient of y,
⇒ $10 - 3r = 1$
⇒ $33r = 9$
⇒ $r = 3$
So, the coefficient of y = ${}^5C_3 \times c^3$
$= 10c^3$

Question 9.
The coefficient of x^{-4} in $(3/2 - 3/x^2)^{10}$ is
(a) 405/226
(b) 504/289
(c) 450/263
(d) None of these

Answer: (d) None of these
Let x^{-4} occurs in (r + 1)th term.
Now, $T_{r+1} = {}^{10}C_r \times (3/2)^{10-r} \times (-3/x^2)^r$
⇒ $T_{r+1} = {}^{10}C_r \times (3/2)^{10-r} \times (-3)^r \times (x)^{-2r}$
Now, we have to find the coefficient of x^{-4}
So, $-2r = -4$
⇒ $r = 2$
Now, the coefficient of $x^{-4} = {}^{10}C_2 \times (3/2)^{10-2} \times (-3)^2$
$= {}^{10}C_2 \times (3/2)^8 \times (-3)^2$
$= 45 \times (3/2)^8 \times 9$
$= (3^{12} \times 5)/2^8$

Question 10.
If n is a positive integer, then $9^{n+1} - 8n - 9$ is divisible by
(a) 8
(b) 16
(c) 32
(d) 64

Answer: (d) 64
Let n = 1, then
$9^{n+1} - 8n - 9 = 9^{1+1} - 8 \times 1 - 9 = 9^2 - 8 - 9 = 81 - 17 = 64$
which is divisible by 64

Let n = 2, then
$9^{n+1} – 8n – 9 = 9^{2+1} – 8 \times 2 – 9 = 9^3 – 16 – 9 = 729 – 25 = 704 = 11 \times 64$
which is divisible by 64
So, for any value of n, $9^{n+1} – 8n – 9$ is divisible by 64

Question 11.
The general term of the expansion $(a + b)^n$ is
(a) $T_{r+1} = {}^nC_r \times a^r \times b^r$
(b) $T_{r+1} = {}^nC_r \times a^r \times b^{n-r}$
(c) $T_{r+1} = {}^nC_r \times a^{n-r} \times b^{n-r}$
(d) $T_{r+1} = {}^nC_r \times a^{n-r} \times b^r$

Answer: (d) $T_{r+1} = {}^nC_r \times a^{n-r} \times b^r$
The general term of the expansion $(a + b)^n$ is
$T_{r+1} = {}^nC_r \times a^{n-r} \times b^r$

Question 12.
In the expansion of $(a + b)^n$, if n is even then the middle term is
(a) $(n/2 + 1)^{th}$ term
(b) $(n/2)^{th}$ term
(c) n^{th} term
(d) $(n/2 – 1)^{th}$ term

Answer: (a) $(n/2 + 1)^{th}$ term
In the expansion of $(a + b)^n$,
if n is even then the middle term is $(n/2 + 1)^{th}$ term

Question 13.
The smallest positive integer for which the statement $3^{n+1} < 4^n$ is true for all
(a) 4
(b) 3
(c) 1
(d) 2

Answer: (a) 4
Given statement is: $3^{n+1} < 4^n$ is

Let n = 1, then
$3^{1+1} < 4^1 = 3^2 < 4 = 9 < 4$ is false
Let n = 2, then
$3^{2+1} < 4^2 = 3^3 < 4^2 = 27 < 16$ is false
Let n = 3, then
$3^{3+1} < 4^3 = 3^4 < 4^3 = 81 < 64$ is false
Let n = 4, then
$3^{4+1} < 4^4 = 3^5 < 4^4 = 243 < 256$ is true.
So, the smallest positive number is 4

Question 14.
The number of ordered triplets of positive integers which are solution of the equation x + y + z = 100 is
(a) 4815
(b) 4851
(c) 8451
(d) 8415

Answer: (b) 4851
Given, x + y + z = 100
where $x \geq 1, y \geq 1, z \geq 1$
Let u = x – 1, v = y – 1, w = z – 1
where $u \geq 0, v \geq 0, w \geq 0$
Now, equation becomes
u + v + w = 97
So, the total number of solution = $^{97+3-1}C_{3-1}$
$= {}^{99}C_2$
= (99 × 98)/2
= 4851

Question 15.
if n is a positive ineger then $2^{3n} – 7n – 1$ is divisible by
(a) 7
(b) 9
(c) 49
(d) 81

Answer: (c) 49
Given, $2^{3n} – 7n – 1 = 2^{3\times n} – 7n – 1$
$= 8^n – 7n – 1$

$= (1 + 7)^n - 7n - 1$
$= \{{}^nC_0 + {}^nC_1\, 7 + {}^nC_2\, 7^2 + \dots\dots + {}^nC_n\, 7^n\} - 7n - 1$
$= \{1 + 7n + {}^nC_2\, 7^2 + \dots\dots + {}^nC_n\, 7^n\} - 7n - 1$
$= {}^nC_2\, 7^2 + \dots\dots + {}^nC_n\, 7^n$
$= 49({}^nC_2 + \dots\dots + {}^nC_n\, 7^{n-2})$
which is divisible by 49
So, $2^{3n} - 7n - 1$ is divisible by 49

Question 16.
The greatest coefficient in the expansion of $(1 + x)^{10}$ is
(a) $10!/(5!)$
(b) $10!/(5!)^2$
(c) $10!/(5! \times 4!)^2$
(d) $10!/(5! \times 4!)$

Answer: (b) $10!/(5!)^2$
The coefficient of xr in the expansion of $(1 + x)^{10}$ is ${}^{10}C_r$ and ${}^{10}C_r$ is maximum for r = 10/ = 5
Hence, the greatest coefficient = ${}^{10}C_5$
$= 10!/(5!)^2$

Question 17.
If A and B are the coefficient of xn in the expansion $(1 + x)^{2n}$ and $(1 + x)^{2n-1}$ respectively, then A/B equals
(a) 1
(b) 2
(c) 1/2
(d) 1/n

Answer: (b) 2
$A/B = {}^{2n}C_n / {}^{2n-1}C_n$
$= \{(2n)!/(n! \times n!)\}/\{(2n - 1)!/(n! \times (n - 1!))\}$
$= \{2n(2n - 1)!/(n(n - 1)! \times n!)\}/\{(2n - 1)!/(n! \times (n - 1!))\}$
$= 2$
So, A/B = 2

Question 18.
$(1.1)^{10000}$ is _____ 1000
(a) greater than

(b) less than
(c) equal to
(d) None of these

Answer: (a) greater than
Given, $(1.1)^{10000} = (1 + 0.1)^{10000}$
$= {}^{10000}C_0 + {}^{10000}C_1 \times (0.1) + {}^{10000}C_2 \times (0.1)^2$ + other +ve terms
$= 1 + 10000 \times (0.1)$ + other +ve terms
$= 1 + 1000$ + other +ve terms
> 1000
So, $(1.1)^{10000}$ is greater than 1000

Question 19.
If n is a positive integer, then $(\sqrt{3}+1)^{2n} + (\sqrt{3}-1)^{2n}$ is
(a) an odd positive integer
(b) none of these
(c) an even positive integer
(d) not an integer

Answer: (c) an even positive integer
Since n is a positive integer, assume n = 1
$(\sqrt{3} + 1)^2 + (\sqrt{3} - 1)^2$
$= (3 + 2\sqrt{3} + 1) + (3 - 2\sqrt{3} + 1)$ {since $(x + y)^2 = x^2 + 2xy + y^2$}
= 8, which is an even positive number.

Question 20.
if $y = 3x + 6x^2 + 10x^3 + \ldots\ldots\ldots.$ then x =
(a) $4/3 - \{(1 \times 4)/(3^2 \times 2)\}y^2 + \{(1 \times 4 \times 7)/(3^2 \times 3)\}y^3 - \ldots\ldots\ldots..$
(b) $-4/3 + \{(1 \times 4)/(3^2 \times 2)\}y^2 - \{(1 \times 4 \times 7)/(3^2 \times 3)\}y^3 + \ldots\ldots\ldots..$
(c) $4/3 + \{(1 \times 4)/(3^2 \times 2)\}y^2 + \{(1 \times 4 \times 7)/(3^2 \times 3)\}y^3 + \ldots\ldots\ldots..$
(d) None of these

Answer: (d) None of these
Given, $y = 3x + 6x^2 + 10x^3 + \ldots\ldots\ldots.$
$\Rightarrow 1 + y = 1 + 3x + 6x^2 + 10x^3 + \ldots\ldots\ldots$
$\Rightarrow 1 + y = (1 - x)^{-3}$
$\Rightarrow 1 - x = (1 + y)^{-1/3}$
$\Rightarrow x = 1 - (1 + y)^{-1/3}$
$\Rightarrow x = (1/3)y - \{(1 \times 4)/(3^2 \times 2)\}y^2 + \{(1 \times 4 \times 7)/(3^2 \times 3!)\}y^3 - \ldots\ldots\ldots..$
